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## THE MECHANISM OF CORONA DISCHARGE FROM A WATER DROPLET

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315:621.359.7

A model is proposed for electrical discharge from a water droplet, which explains the experimentally observed formation of a sharp projection at the top of the droplet and emission therefrom of highly dispersed charged droplets and ions.

1. The first studies of electrical discharge from a water droplet were performed at the start of this century [1]. After the appearance of [2] such a discharge was termed "corona," although from the data obtained in [2] it follows that such a term is incorrect (which was noted in [3]). This incorrect terminology has led to the treatment of the still incompletely studied mechanism of discharge from a water drop from the viewpoint of conventional corona discharge from a metallic point [4]. Meanwhile the study of the real mechanism of discharge from a drop of electrically conductive liquid is of significant interest because of its applications in various physical and technical problems: from power generation [4] and study of the mechanisms of explosive emission and electrode erosion in high power arcs [5] to the theory of natural storm electricity [6]. The present study was undertaken to meet this need.

It is well known that in a sufficiently intense electric field a water drop first extends into an ellipsoid of revolution oriented along the field [7], after which sharp projections appear at the ends, from which highly dispersed droplets which flow weakly in darkness are emitted [1-4, 6, 8]. The dimensions of these droplets decrease with increase in field, and finally at a sufficiently high field, droplet emission is replaced by emission of ions [1, 6, 8]. The visual characteristics of a discharge from a water drop very much recall a corona discharge from a metal point. This is evident from Figs. 1 and 2. The external similarity of the two processes is obvious. But careful examination of experimental data on the discharge from a water drop [1-4, 6, 8] shows that it cannot be identified with the discharge from the metal point: their mechanisms differ. But before discussing the mechanism of the drop discharge we should first consider the question of why the sharp projections from which the discharge commences appear upon the drop.

The classical studies of Tonks [9] and Taylor [10] on this question did not in fact provide an answer. Tonks showed that formation of such projections is possible, or to speak more precisely, does not contradict known physical laws. Taylor simply postulated existence of equilibrium projections of strictly conical form with aperture angle of the cone constant for a given liquid, and commencing from this postulate, sought the external field at which existence of that form was possible. And apparently, he was incorrect. Careful examination of available photographs of such projections, including those cited by Taylor [8, 10-13], shows that their form is not precisely conical, but is more like a half of a pseudosphere [14]: aside from the close vicinity of the top itself, the Gaussian curvature of the projection is negative everywhere. This is especially evident in the photograph

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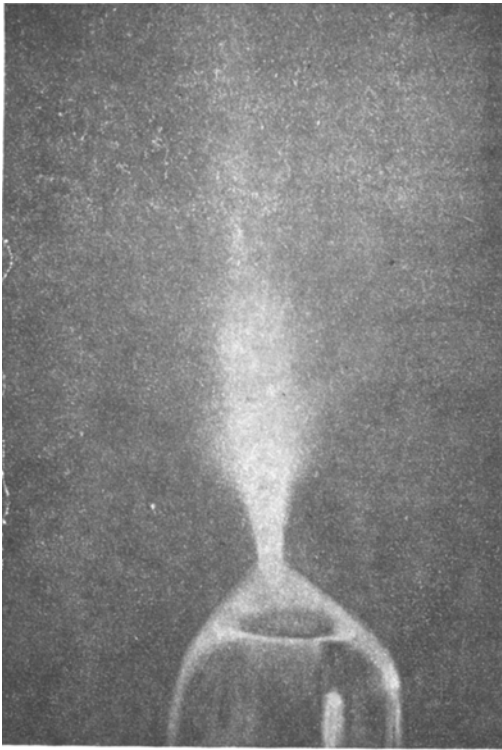


Fig. 1



Fig. 2

Fig. 1. Photograph of electrical discharge from a water drop 2.5 mm in diameter upon application of positive potential of  $\approx 10$  kV.

Fig. 2. Photograph of corona discharge from metallic point upon application of positive potential of  $\approx 10$  kV.

presented in [13] (p. 247). From this photograph, which recorded several simultaneously existing projections, it follows that if we can even speak of their aperture angle, that quantity is not constant for a given liquid. But in that case all the constructions based on Taylor's postulate are erroneous. This conclusion explains the unsuccessful attempts to reconcile Taylor's theoretical views with experimental data in which the effect of projections on a liquid surface is significant: in liquid metal ion sources [11] and in electrodynamic atomization of liquids [15]. On the basis of this fact, [11] raised doubt as to the equilibrium nature of the projections. And in our opinion, this doubt is fully justified, since Zeleny [16] also thought that the electric current flowing through the drop played an important role in formation of the projections. As will be shown below, consideration of this effect will explain the appearance of the projections, and thus opens a path for detailed study of the mechanism of discharge from a water drop.

2. Let a drop of nonideally conductive liquid be located in air within a constant homogeneous electric field  $E_0$ . If the field intensity is not too great and instability of the drop surface exists, then, as is well known [7], the equilibrium form of the drop will be an ellipsoid of revolution extended along the field. According to Reilly's theory [17] instability of the liquid surface implies exponential growth of certain modes of capillary waves which exist in the droplet in view of the thermal motion of liquid molecules [18]. This occurs at a value of external field  $E_*$  related to the surface tension coefficient  $\sigma$  and the radius of the original spherical drop  $R_0$  by the phenomenological expression [10]

$$4\pi\epsilon_0 E_*^2 R_0 \sigma^{-1} \geq 2,62. \quad (1)$$

It is natural to expect that upon development of instability the first capillary wave modes to increase will be those symmetric about the direction of the field. This then causes an exponentially increasing axisymmetric projection to appear at the end of the drop turned toward the field. But within the framework of linear theory exponential amplitude growth is guaranteed only until the mode amplitude becomes of the order of the wavelength. Linear theory is unable to predict the further fate of the unstable mode, but numerical calculations [15] show that at some finite amplitude mode growth terminates. In the final outcome the

projection at the end of the ellipsoidal drop stabilizes after reaching some finite value.

3. The appearance of a projection at the end of the drop leads to redistribution over the drop section of the electric current which flows within it because of the field  $E_0$ , which will be homogeneous in an ellipsoidal drop [19]. In pure (deionized) water the current is caused by proton transitions over hydrogen bonds [20], i.e., it is a flow of charged particles all of the same sign, which exerts an effect upon the medium, in which a certain pressure propagates, and thus, upon the drop-external medium boundary (i.e., we have an analog of dynamic pressure). But, as can easily be seen, this pressure has the same dependence upon coordinate as does the pressure of the external field [19]:

$$P_E = \frac{\epsilon_0 E^2}{2} = \kappa \frac{\epsilon_0 E_0^2}{2} \frac{1 - \frac{\rho^2}{b^2}}{1 + \frac{e^2}{1 - e^2} \frac{\rho^2}{b^2}}, \quad (2)$$

where  $\kappa$  is a coefficient depending on the eccentricity, electrical conductivity, and dielectric permittivity of the drop and external medium. In fact, the pressure of the constant current on the ellipsoid surface  $\sim \cos^2 \theta$ , where  $\theta$  is the angle between the direction of the  $H^+$  ion motion creating the current to the normal at the given point of the ellipsoid surface. Taking into consideration that  $\tan \zeta = -\dot{z}(\rho)$ , where  $\dot{z}$  is the tangent to the meridional section of the ellipsoid  $z = z(\rho): z = a\sqrt{1 - \rho^2/b^2}$ , one can easily write an expression for the pressure produced by the current:

$$P_T = P_+ \frac{1 - \frac{\rho^2}{b^2}}{1 + \frac{e^2}{1 - e^2} \frac{\rho^2}{b^2}}, \quad (3)$$

i.e., we obtain the same function of coordinates as for the field pressure, Eq. (2). And this means that consideration of Eq. (3) cannot lead to a qualitative change in drop form. And since  $P_+ \ll \epsilon_0 \kappa E_0^2/2$ , then  $P_T$  far from the state of instability can in general be neglected. The picture changes when a projection appears at the top of the drop, even if it be small. For simplicity we will approximate the form of the projection by one half of an oblate ellipsoid of revolution. As is well known [19], field intensity in the vicinity of the peak of an ellipsoid both outward and inward is higher than the homogeneous field  $E_0$  far from the ellipsoid by a factor of  $\alpha$  times, where  $\alpha$  is determined by the eccentricity (for a sphere, for example,  $\alpha = 3$ ). The same is true of an ellipsoidal projection atop an ellipsoidal drop: appearance of the projection produces a local increase in field as compared to that existing originally. In the final reckoning the current flowing through the drop to its top beneath the projection increases, while far from the projection (within the drop) it remains as before (the projection is assumed small, and its development causes only local changes). This means that the ion flow rate beneath the projection increases somewhat. Since the field beneath a projection in the form of an ellipsoid of revolution is homogeneous [19], just as in the main volume of the ellipsoidal drop, with consideration of the effect of the projection the flow in the drop can be modeled by a "jet with satellite flow" in hydrodynamic representation. In this case the distribution of ion velocity over distance to the axis of symmetry of the drop will have the form (see, for example, [21]):

$$V - V_0 = C \left(1 + \frac{\rho^2}{\beta^2}\right)^{-1}. \quad (4)$$

This means that at the top of the drop near the axis of symmetry an ion current pressure in addition to that of Eq. (3) will act:

$$P_* = \frac{nm}{2} (V - V_0)^2 = \frac{\frac{nmC^2}{2}}{\left(1 + \frac{\rho^2}{\beta^2}\right)^2} \equiv \frac{B}{1 + \frac{\rho^2}{\beta^2}}. \quad (5)$$

We will seek the form of the drop under these conditions using the technique employed for analysis of conditions of development of toroidal figures of equilibrium liquid masses subjected to the action of capillary forces [22].

4. In view of the axial symmetry of the problem developed it will be sufficient to limit ourselves to the first quadrant of the meridional section of the drop, then  $\rho \equiv x$ .

On the free surface of the drop the condition of equilibrium of all forces has the form

$$P_i + P_\sigma = P_E + P_0 + P_\tau + P_*, \quad (6)$$

where in the first quadrant [14, 22]:

$$P_\sigma = -\frac{\sigma}{x} \frac{d}{dx} \frac{xz}{\sqrt{1+z^2}}. \quad (7)$$

Substituting Eqs. (2), (3) and (5), (7) in Eq. (6) and transforming to dimensionless variables:

$$\begin{aligned} \eta &= \frac{z}{b}; \quad \xi = \frac{x}{b}; \quad q = \frac{b}{\beta}; \quad \delta = \frac{bB}{2\sigma}; \\ c_0 &= \frac{(P_0 - P_i) b}{2\sigma}; \quad h = \left( \frac{\varepsilon_0 E_0^2 \alpha b}{4\sigma} + \frac{P_+}{\sigma} \right); \\ \gamma &= \frac{e^2}{1 - e^2}, \end{aligned} \quad (8)$$

we arrive at

$$-\frac{1}{\xi} \frac{d}{d\xi} \frac{\xi \dot{\eta}}{\sqrt{1+\eta^2}} = 2c_0 + 2h \frac{1+\xi^2}{1+\gamma\xi^2} + \frac{2\delta}{(1+\xi^2 q^2)^2}. \quad (9)$$

Integrating Eq. (9) one time, we obtain

$$-\frac{\dot{\eta}}{\sqrt{1+\eta^2}} = c_0 \xi + \frac{h}{\gamma} \left[ \frac{\gamma+1}{\gamma} \frac{\ln(1+\gamma\xi^2)}{\xi} - \xi \right] + \frac{\delta}{q^2} \frac{1}{\xi(1+q^2\xi^2)} + \frac{c_1}{\xi}, \quad (10)$$

where  $c_1$  is an integration constant. For any value of  $\dot{\eta}$  the expression on the left side of Eq. (10) will be finite, while as  $\xi \rightarrow 0$  the expression on the right increases because of the last two terms. Requiring that the sum of these remain finite as  $\xi \rightarrow 0$ , we obtain  $c_1 = -\delta/q^2$ . Finally we obtain

$$-\frac{\dot{\eta}}{\sqrt{1+\eta^2}} = c_0 \xi + \frac{h}{\gamma} \left[ \frac{\gamma+1}{\gamma} \frac{\ln(1+\gamma\xi^2)}{\xi} - \xi \right] - \frac{\delta\xi}{1+q^2\xi^2} \equiv \psi(\xi) \quad (11)$$

or

$$\dot{\eta} = -\frac{\psi}{\sqrt{1-\psi^2}}. \quad (12)$$

The solution of this equation then determines the form of the meridional section of the drop in the first quadrant. In view of the cumbersomeness of the expression obtained Eq. (12) must be integrated numerically. But the information of interest to us can be obtained directly from Eqs. (11), (12) without doing that.

5. We consider that if we drop the last term in Eq. (11) and substitute the abbreviated function  $\psi$  thus obtained in Eq. (12), that equation determines the equilibrium drop form in the field, which according to the above, is ellipsoidal. The basic result of [15] can be reduced to the statement that under the action of only capillary forces and pressure forces of an external electrostatic field it is impossible to obtain a sharp projection atop the drop. This means that if consideration of current pressure can explain the appearance of the projection, then in Eq. (11) this projection is defined by the last term. And if we drop the first two terms in Eq. (11) and retain only the last, i.e., take

$$\psi(\xi) = -\frac{\delta\xi}{1+q^2\xi^2}, \quad (13)$$

then the solution of Eq. (12) gives the form of the projection which is imposed on the equilibrium ellipsoid defined by the terms dropped. In this case Eq. (12) takes on the form

$$\dot{\eta} = \frac{\delta\xi}{\sqrt{(1+q^2\xi^2)^2 - \delta^2\xi^2}}$$

and is easily integrated

$$\eta(\xi) = \frac{\delta}{2q^2} \operatorname{arsh} \frac{2q^2\xi^2 + \left(2 - \frac{\delta^2}{q^2}\right)}{\sqrt{2\frac{\delta^2}{q^2} - \frac{\delta^4}{q^4}}} \Big|_{\xi}^1. \quad (14)$$

Figure 3 shows  $\eta(\xi)$  for various values of  $q$  and  $\delta$ . It is evident from Eq. (14) that the amplitude of this projection has a value

$$L \equiv \frac{\delta}{2q^2} \left( \operatorname{arsh} \frac{2q^2 + 2 - \frac{\delta^2}{q^2}}{\frac{\delta}{q} \sqrt{2 - \frac{\delta^2}{q^2}}} - \operatorname{arsh} \sqrt{2\frac{q^2}{\delta^2} - 1} \right), \quad (15)$$

with characteristic width

$$S \sim q^{-1}. \quad (16)$$

6. For small  $\xi$ , where  $\xi \ll 1/q$ , i.e., in the vicinity of the tip of the projection, in place of Eq. (13) we have approximately:  $\psi(\xi) = -\delta\xi$ . The solution of Eq. (12) in this case is quite simple:  $\eta^2 + \xi^2 = \delta^{-2}$ , i.e., near the tip the projection is practically spherical and the field within the projection beneath the tip is constant and homogeneous [19]. Thus, we return to the original problem. In fact, if the drop instability condition in the external field, Eq. (1), is satisfied in the original drop, then it will be satisfied even more so for the projection atop the drop with its smaller radius of curvature. Let the drop potential  $\varphi$  remain constant. Then the field intensity at a given point of the surface is defined by the expression  $E = \varphi/R$ , where  $R$  is the mean radius of curvature at the given point [14]. Condition (1) for the tip of the projection then takes on the form

$$E^2 \frac{R}{\sigma} = \frac{\varphi^2}{R^2} \frac{R}{\sigma} = \frac{\varphi^2}{R\sigma} \geq \text{const.}$$

It is evident that with decrease in the radius of curvature the left side of this inequality increases. This means that it is satisfied even more for the tip of the projection if it was satisfied for the original drop. Then because of capillary wave instability [9] there appears again on the projection a smaller projection, under which the conduction current pressure, external field, and capillary forces will act. In the final outcome the projection takes on the form of Eq. (14) with different coefficient values. The process is then repeated anew, i.e., the projection grows. Meanwhile the height of each subsequent step must exceed the previous one, since the field intensity beneath the projection increases with increase in its mean curvature, so that the current density and its pressure on the projection increases and according to Eq. (15) the height of the corresponding step must increase. According to Eq. (16) the width must decrease. Finally the form tends to one half of a pseudosphere [14], as follows from the experimental photographs noted above, but is especially evident in the photographs of [11, 13] or the photograph of the water drop in an external field (see Fig. 4) obtained by the present authors in experiments similar to those of Zeleny [4, 16].

7. As was noted above, it has been observed experimentally that with increase in the external field the tip of the projection on the liquid surface first begins to emit charged liquid droplets [4, 6, 8], the size of which decreases with increase in field, followed by ion emission [5, 6, 8]. Visually the entire process appears like a corona discharge from the water drop [5]. However it is obvious that the physical mechanisms of this discharge may differ greatly from those involved in a corona discharge from a metallic point. We will consider this question in greater detail.

It is clear a priori that equilibrium emission of charged droplets by the projection will be found if only the characteristic time for electrical relaxation in the field  $E$  of the large drop with dielectric permittivity  $\epsilon$ , charge carrier mobility  $u$ , and linear size  $l$ :  $\tau = \epsilon l / uE$  is less than the time for droplet breakoff from the tip of the projection, which will be of the order of the period of oscillation of the fundamental mode of the detached droplet of density  $d$  and radius  $r$  [17]:  $t = 2\pi\sqrt{dr^3/8\sigma}$ , which leads to a limitation on the minimum size of the droplets emitted:

$$r_* > \left\{ \frac{2\sigma}{d} \left( \frac{\epsilon l}{\pi u E} \right)^2 \right\}^{1/3}. \quad (17)$$

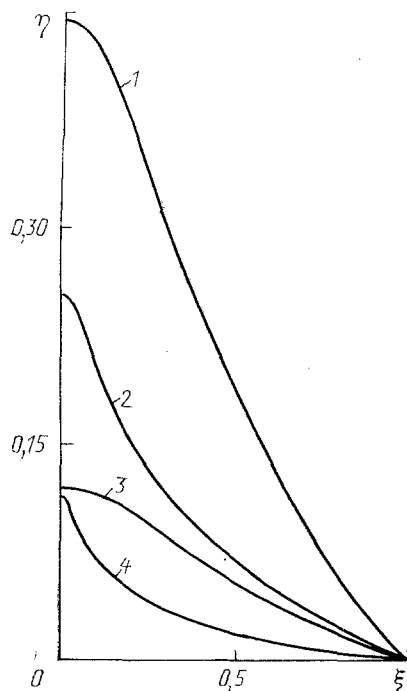


Fig. 3

Fig. 3. Calculated forms of emitter projection: 1)  $q^2 = 10$ ;  $\delta/q = 1$ ; 2) 100 and 1; 3) 10 and 0.1; 4) 1000 and 1.

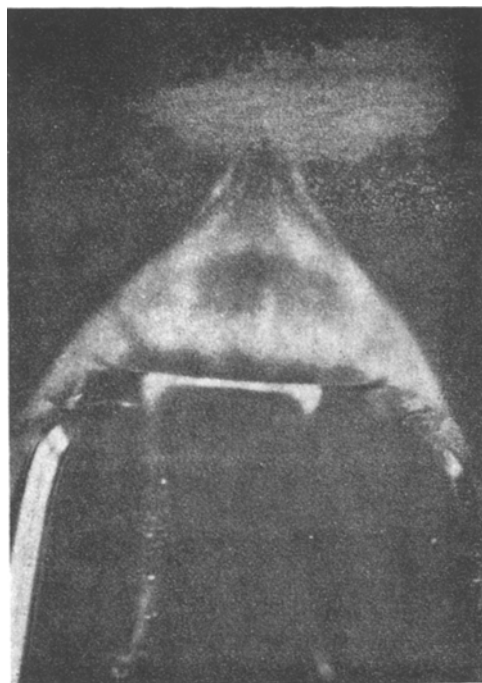


Fig. 4

Fig. 4. Photograph of emitting projection atop drop appearing upon ignition of electrical discharge.

For  $E = 1.5 \cdot 10^5$  V/cm,  $u = 350$  cm<sup>2</sup>/Ω,  $l = 1$  mm,  $\epsilon = 80$ ,  $\sigma = 7 \cdot 10^{-4}$  N/cm, condition (17) takes on the form  $r_* > 7 \cdot 10^{-5}$  cm. The mechanism of droplet detachment from the projection tip is apparently the same as in the case of detachment of a water drop from the end of a capillary tube under the force of gravity, i.e., the external field  $E$  acting on the charge of the projection tip  $Q$  tends to remove the drop while the capillary force  $2\pi r \sigma$  retains it. When the electrical force exceeds the capillary, droplet detachment occurs. However this may all be illustrated by a simplified model of the phenomenon.

According to generally accepted physical concepts of charge distribution on the surface of a conductor of arbitrary form, the maximum charge density will be on convex portions of the conductor with maximum mean and positive Gaussian surface curvature, with minimum charge in depressions where the Gaussian curvature of the surface is negative. In the model under consideration the Gaussian curvature of all points of the projection with the exception of its tip, which may be approximated by a hemisphere, is negative. In connection with this we assume that on the projection charge exists only on the hemispherical tip, where in the equilibrium state it is distributed over the surface with a density  $\Sigma = 3E \cos \theta / 4\pi$  [19] (p. 30). Then the total charge of the projection will equal:  $Q = 0.75 \cdot E \cdot r^2$ . Requiring that the force detaching the droplet from the projection  $QE$  exceed the capillary force restraining the droplet  $2\pi r \sigma$ , we obtain another limitation on the minimum size of the droplets detached

$$r_* > \frac{8\pi\sigma}{3E^2} \quad (18)$$

In this condition the strong dependence of  $r_*$  on  $E$  is interesting, allowing explanation of the experimental fact of reduction in size of the droplets emitted with increase in the applied field. For the parameter values chosen Eq. (18) yields:  $r_* > 2 \cdot 10^{-3}$  cm, while for increase in  $E$  by an order of magnitude the limitations on  $r_*$  given by Eqs. (17), (18) coincide.

Disruption of Eq. (17) implies that electrical charge does not reach the projection due to the finite value of the electrical conductivity and the total charge of the projection tip is less than the equilibrium value, which raises doubt as to the satisfiability of condition (18).

8. But even before the finite velocity of charge transfer in the nonideally conducting droplet limits emission of charged droplets from the projection, another mechanism for the

removal of charge from the tip sets in: auto-ion emission [8, 11]. As soon as the local electric field intensity at the tip of the projection reaches a value of  $\sim 10^8$  V/cm, field evaporation of  $H^+$  ions begins. And a field of  $\sim 10^8$  V/cm for the potentials characteristic of Zeleny's [4] and English's [5] experiments of  $\sim 10^4$  V is achieved at the tip of the projection even at a radius of curve of  $\sim 10^{-4}$  cm.

The fact of charge removal from the drop by ions has been noted more than once in experiments (see, for example, [8]). Apparently such ion emission from the tip of the projection is the cause of the fan-shaped scintillation observed by Zeleny [4], the special form of corona discharge from a water drop. In fact, the free path length for an ion before collision with a neutral molecule in air at atmospheric pressure is  $\sim 10^{-5}$  cm [23], and the energy acquired over the free path length by an  $H^+$  ion in a field of  $\sim 10^8$  V/cm, which develops at the tip of the projection, will be  $\sim 10^3$  eV. A proton with such energy will ionize air and excite a molecule at a distance of  $\sim 10^{-4}$  cm in the vicinity of the tip [13], which for a main drop with diameter of  $\sim 1$  mm and currents of  $\sim 1$   $\mu$ A [16] will produce  $\sim 10^{15}$  excited molecules per second [8]. Over a time of  $10^{-7}$  sec about  $10^8$  excited molecules will appear, the radiation of which, according to the estimates presented in [23] (p. 141), will generate  $\sim 10$  photoelectrons at a distance of  $\sim 1$  cm from the tip. In the inhomogeneous electric field  $\geq 10^4$  V/cm between the drop and the plane electrode these electrons will form an avalanche directed toward the drop, i.e., a corona discharge supported by photoionization will exist.

9. In conclusion it should be noted that the proposed model of an electrical discharge from a conductive liquid drop can be used to study the physical mechanisms of operation of ion colloid reactive engines [24], where emission of charged droplets by a conductive liquid is significant, in the theory of storm electricity involving St. Elmo's fire [25], the model of which has been based on the "fan" scintillation from a water drop observed by Zeleny [1], as well as in designing liquid metal ion sources [11] and electrodynamic liquid atomizers [15].

#### NOTATION

$x, z$ , dimensional Cartesian coordinates;  $b, a$ , minor and major semiaxis of ellipsoid of rotation oriented along axis OZ;  $e$ , ellipsoid eccentricity;  $\xi = x/b$  and  $\eta = z/b$ , dimensionless coordinates;  $R_0$ , spherical drop radius;  $R$ , mean radius of surface curvature;  $r$ , radius of small emitted droplet;  $r_x$ , minimum radius of emitted droplet;  $\rho$ , distance from axis of symmetry of ellipsoid;  $\zeta$ , slope of tangent to meridional section of surface;  $\theta$ , orientation angle of external normal to drop surface relative to external homogeneous electric field  $E_0$ ;  $E_x$ , critical value of external field at which drop becomes unstable;  $E$ , electric field intensity at drop surface;  $\alpha$ , field intensification coefficient beneath curved surface compared to field beneath plane surface;  $\sigma$ , surface tension coefficient of water;  $\epsilon_0$ , dielectric constant;  $\epsilon$ , dielectric permittivity of water;  $n$ , ion concentration;  $m$ , ion mass;  $V$ , velocity of ion motion in water produced by electric current;  $V_0$ , unperturbed velocity of ion motion under action of homogeneous electric field in ellipsoidal drop;  $\beta$ , characteristic transverse dimension of projection atop drop;  $d$ , density of water;  $Q$ , charge at tip of projection on drop;  $\Sigma$ , electrical charge surface density on drop;  $\tau$ , characteristic time for drop electrical relaxation;  $\varphi$ , electrical potential of drop;  $P_1$ , atmospheric pressure;  $P_0$ , Laplace pressure beneath curved drop surface;  $P_E$ , electrostatic pressure of external field on drop surface;  $P_T$ , pressure of current on drop surface, when ellipsoidal form is not distorted by any disturbance;  $P_+$ , proportionality coefficient in expression for current pressure;  $P_x$ , current pressure disturbance due to appearance of projection atop ellipsoid of revolution;  $B$ , proportionality coefficient with dimensions of pressure;  $C$ , proportionality coefficient with dimensions of velocity;  $\kappa$ , dimensionless proportionality coefficient;  $c_1$ , integration constant;  $P_0$ , internal pressure in drop;  $u$ , ion mobility;  $l$ , characteristic linear dimension of drop;  $\psi(\xi)$ , function of dimensionless coordinate  $\xi$ ;  $q = b/\beta$ ;  $v = e^2/(1-e^2)$ ;  $\delta = bB/2\sigma$ ;  $c_0 = (P_0 - P_1)b/2\sigma$ ;  $h = \epsilon_0 E_0^2 \alpha b/4\sigma + P_+/\sigma$ , dimensionless parameters;  $L, S$ , amplitude and width of projection formed on top of equilibrium spheroid.

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#### EFFECT OF ELECTRODE CONFIGURATION ON EFFICIENCY OF INDUCTION

#### ELECTRIFICATION OF DROPLETS

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Numerical calculation of the electric field in an induction electrification unit is used to determine the charges of droplets formed for various axisymmetric electrode systems.

In electrodroplet-jet equipment the charge of the droplets is the basic parameter controlling transverse deviations of drops in the chain and, thus, formation of the image on the substrate [1]. Therefore calculation of this parameter is an important stage in the design of such devices. Theoretical calculations are based on determination of the electric field in the interelectrode gap. Analytical calculations usually make use of a number of assumptions for which quantitative estimates are lacking, as a result of which they are uncontrolled, and in a number of cases, incorrect. Among these assumptions are: that the droplet charge is determined by a section of the jet the length of which is equal to the wavelength of the disturbance; neglect of electrode edge effects; neglect of charge redistribution on the electrode surfaces due to interaction with jet charges and detached droplets; use of the thin bar approximation for the jet, which produces the largest error near its end (i.e., the spot of droplet formation), etc.

The present study has as its goal the calculation of the electric field in the interelectrode space of an axisymmetric charging system in an electrodroplet-jet device with its subsequent use to calculate the charges upon the droplets formed. Special attention was

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